Common Distributions
~ summary

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"What is $\mathbb{P}$ of getting x success in n trials?"

$$\mathbb{P}(X=x\mid n,p)=\boxed{\binom{n}{x}}p^x(1-p)^{n-x}$$

ways of ordering x success out n

n Trials

$$\mathbb{P}(X=x\mid p)=p^x(1-p)^{1-x}$$

$$X=\begin{cases}1 & \text{success} \quad p\\ 0 & \text{failure} \quad 1-p\end{cases} \qquad 0\le p\le 1$$

only 2 outcomes 1 trial

Bernoulli

Binomial

$$\mathbb{P}(X=x\mid N)=1/N$$

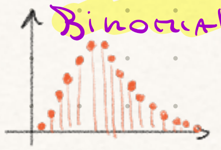
Uniform

Discrete

r Success

"how many trials do we need for r success?"

Negative Binomial

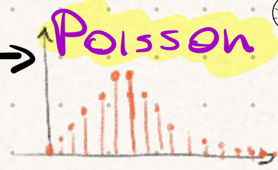


$y :=$ Trials at which the r_{th} success occurs

$$\mathbb{P}(y=y\mid r,p)=\binom{y-1}{r-1}p^r(1-p)^{y-r}$$

$r=1$

Poisson



$$\mathbb{P}(X=x\mid\lambda)=\frac{e^{-\lambda}\lambda^x}{x!} \qquad x=0,1,\dots$$

The most used for waiting times

Like $X :=$ # calls in a minute

$\lambda :=$ $\mathbb{E}$ value of calls

As $t\to 1$ minute $\mathbb{P}\downarrow 0$

$t\in\mathcal{B}(0)$ ~ almost uniform distribution i.e. same probability

Geometric

$$\mathbb{P}(X=x\mid p)=p(1-p)^{x-1}$$

$X :=$ Trial at which the first success occurs

Memoryless

$$\mathbb{P}(X>s\mid X>t)=\mathbb{P}(X>s-t)$$

Waiting for a success

Used to model "lifetimes", "Time until failure"

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Continuous

Normal

Let $X \sim \mathcal{N}(\mu, \sigma^2)$

Properties

① $(X+b) \sim \mathcal{N}(\mu+b, \sigma^2)$ translation

② $a \cdot X \sim \mathcal{N}(ab, a^2\sigma^2)$

③ Normal & Binomial: If n is big & p not close to 0 or 1 then

$X \sim Bin(n,p) \approx Y \sim \mathcal{N}(np, np(1-p))$

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

Normalize Area — Center at μ — symmetry

Beta

$$f(x|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad x \in (0,1) \quad \alpha>0, \beta>0$$

many shapes → Great to approx data

symmetric → more concentrated $\alpha \to \infty$

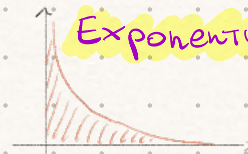
$\alpha = \beta = 1$ uniform distribution

Beta function $\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$

Gamma function relationship

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Exponential



$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

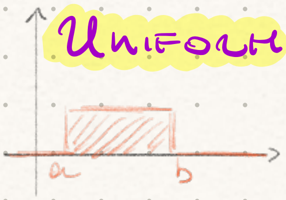
- Exponential Families
- memoryless
- particular case of Gamma distribution
- analogue to Geometric Distribution

A pdf/pmf is called an Exponential Family iff

$$f(x|\theta) = h(x)c(\theta)\exp\left(\sum_{i=1}^{k} w_i(\theta)t_i(x)\right)$$

$h(x) \geq 0$, $c(\theta) \geq 0$, w_i & t_i realvalued

Uniform



$$f(x|a,b) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$

Gamma

$\alpha = 1$

$$f(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$x \in (0, \infty)$, $\alpha, \beta > 0$ (shape, rate)

Gamma function

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt$$

$\Gamma(\alpha+1) = \alpha\Gamma(\alpha) \Rightarrow \Gamma(n) = (n-1)!$

memoryless

If $X \sim Gamma(\alpha,\beta)$

$\mathbb{P}(X \leq x) = \mathbb{P}(Y \geq \alpha)$

$Y \sim Poisson(x/\beta)$